Vypočítajte limity:

7.) $\lim_{x \to -2} \frac{3x+6}{x^3+8}$ 8.) $\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

9.) $\lim_{x \to 0} \left[ \frac{\sin x}{x} + \frac{\sin 3x}{x} + \frac{\sin 5x}{x} \right]$ 10.) $\lim_{x \to 0} \frac{\sin x}{\sqrt{1-\cos x}}$

Riešenie:

7.) $$\lim_{x \to -2} \frac{3x+6}{x^3+8} = \lim_{x \to -2} \frac{3(x+2)}{(x+2)(x^2-2x+4)} = \lim_{x \to -2} \frac{3}{x^2-2x+4} =$$

$$= \frac{3}{(-2)^2-2(-2)+4} = \frac{3}{12} = \frac{1}{4}$$

8.) $$\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} = \lim_{x \to 0} \frac{2+x-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)} =$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

9.) $$\lim_{x \to 0} \left[ \frac{\sin x}{x} + \frac{\sin 3x}{x} + \frac{\sin 5x}{x} \right] = \lim_{x \to 0} \left[ \frac{\sin x}{x} + \frac{3\sin 3x}{3x} + \frac{5\sin 5x}{5x} \right] = 1 + 3.1 + 5.1 = 9$$

10.) $$\lim_{x \to 0} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x \to 0} \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \lim_{x \to 0} \sqrt{\frac{1-\cos^2 x}{1-\cos x}} = \lim_{x \to 0} \sqrt{\frac{(1-\cos x)(1+\cos x)}{1-\cos x}} =$$

$$= \lim_{x \to 0} \sqrt{1+\cos x} = \sqrt{1+\cos 0} = \sqrt{1+1} = \sqrt{2}$$